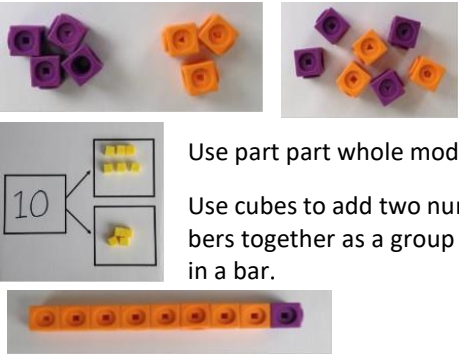
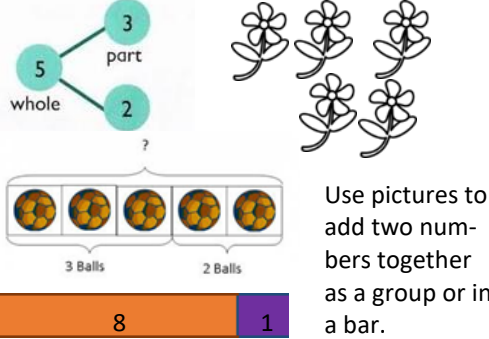
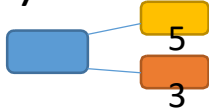
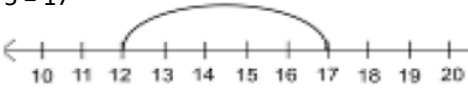
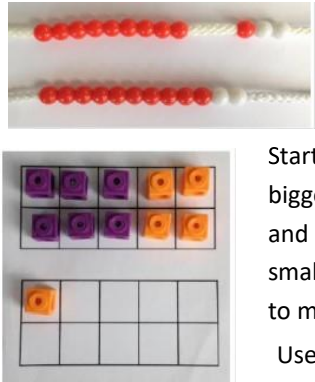
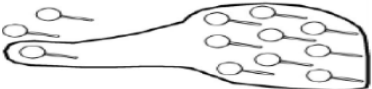
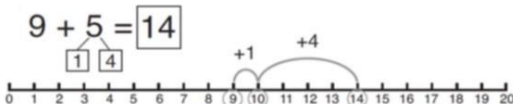
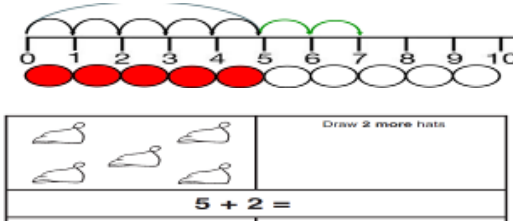


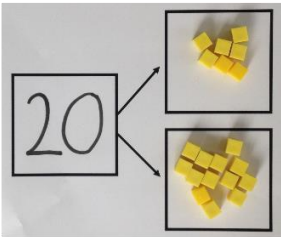
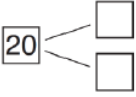
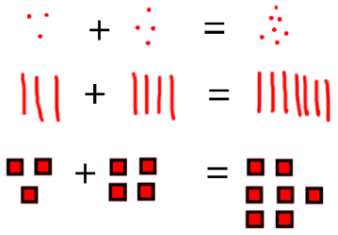
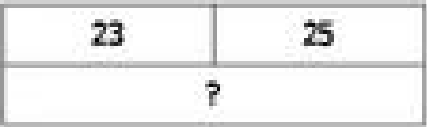
Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	 Use pictures to add two numbers together as a group or in a bar.	$4 + 3 = 7$  Use the part-part whole diagram as shown above to move into the abstract. $10 = 6 + 4$
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	$12 + 5 = 17$  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	$5 + 12 = 17$ Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. <i>This is an essential skill for column addition later.</i>	 Start with the bigger number and use the smaller number to make 10. Use ten frames.	$3 + 9 =$  Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	$7 + 4 = 11$ If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 Draw 2 more hats $5 + 2 =$	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

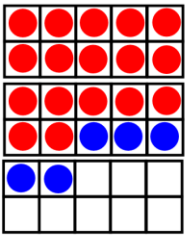
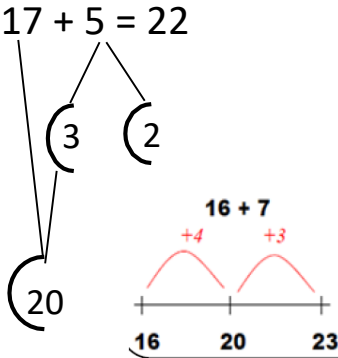
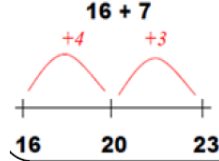
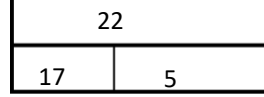
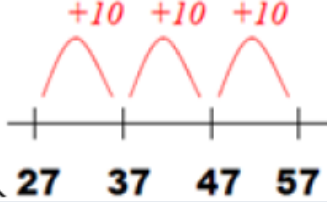
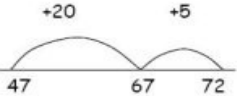
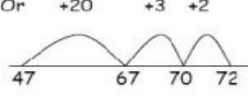
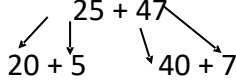
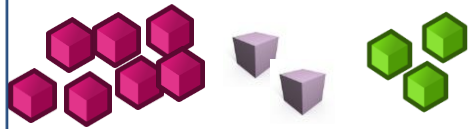
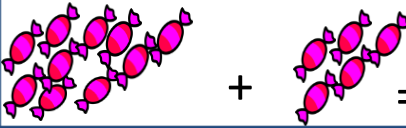
Y1

ADDITION +

Y2

ADDITION +

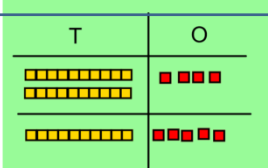
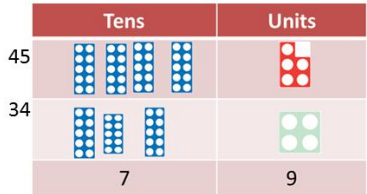
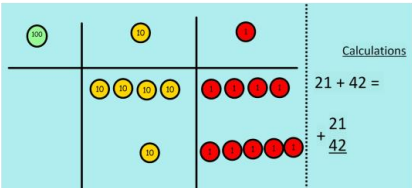
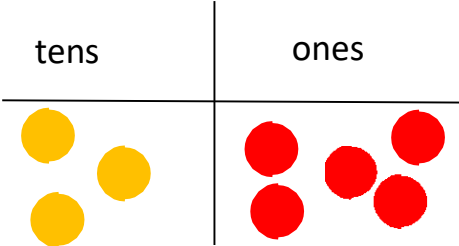
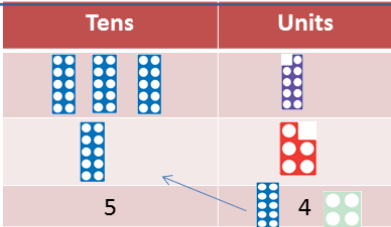
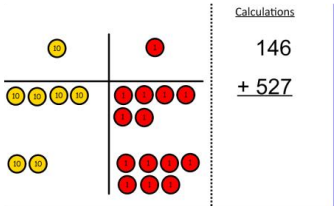
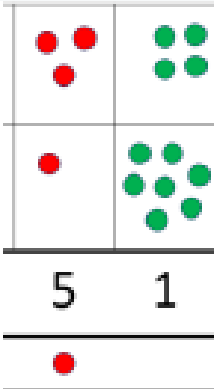
Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$ $30 + 50 = \underline{\hspace{2cm}}$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <i>Part part whole</i>	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts	$\text{L-shaped blocks} + \text{L-shaped blocks} = \text{L-shaped blocks}$ $\text{Vertical bars} + \text{Vertical bars} = \text{Vertical bars}$	 Children draw representations of H,T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$

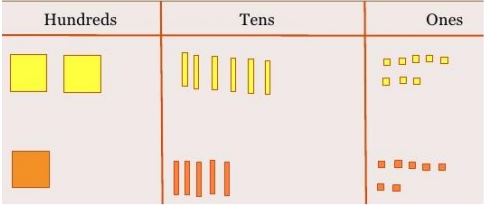
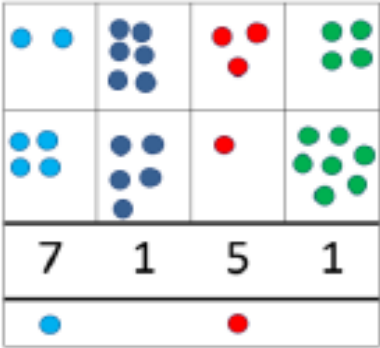
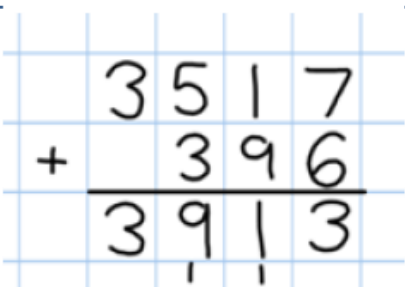
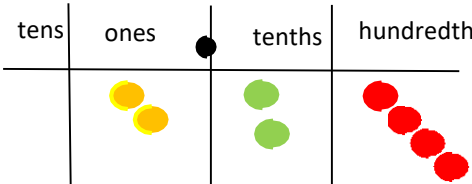
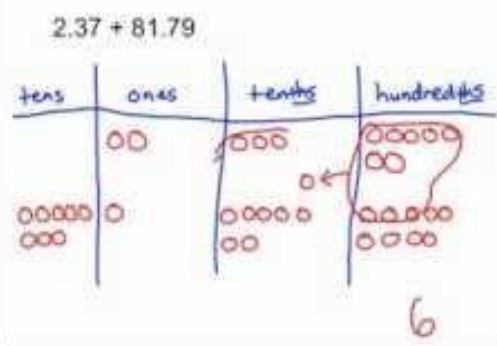
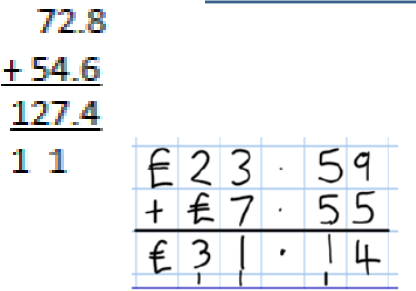
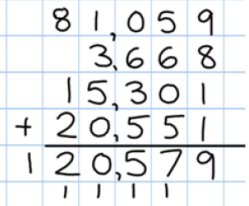
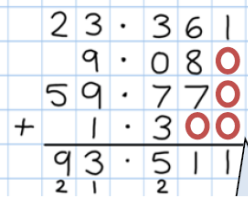
Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	 $17 + 5 = 22$ Use ten frame to make 'magic ten' Children explore the pattern. $17 + 5 = 22$ $27 + 5 = 32$	Use part part whole and number line to model. $17 + 5 = 22$  $16 + 7$ 	$17 + 5 = 22$ Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ 
Add a 2 digit number and tens	 $25 + 10 = 35$ Explore that the ones digit does not change	$27 + 30$  $27 \quad 37 \quad 47 \quad 57$	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$
Add two 2-digit numbers	 Model using dienes, place value counters and numicon	 $47 \quad 67 \quad 72$ Or  $47 \quad 67 \quad 70 \quad 72$ Use number line and bridge ten using part whole if necessary.	$25 + 47$  $20 + 5$ $40 + 7$ $20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 10 then add third digit	 Regroup and draw representation.  $47 + 8 = 55$	$4 + 7 + 6 = 10 + 7$ 10 $= 17$ Combine the two numbers that make/ bridge ten then add on the third.

Y3

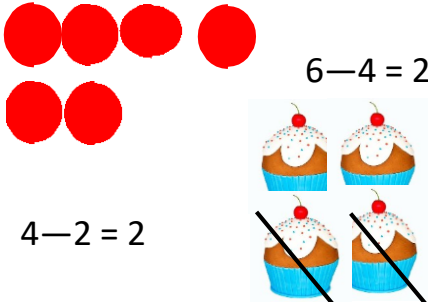
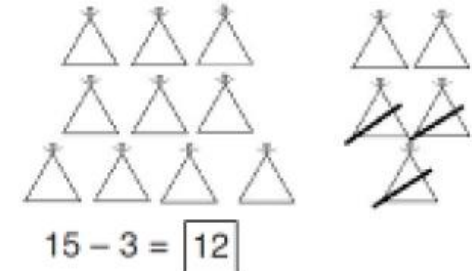
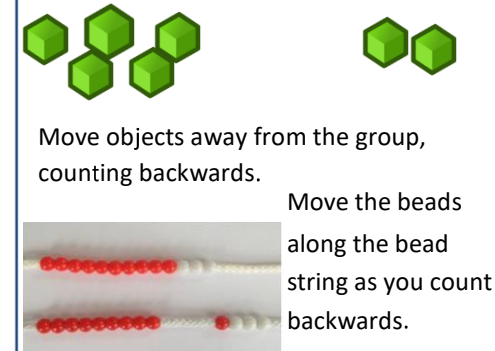
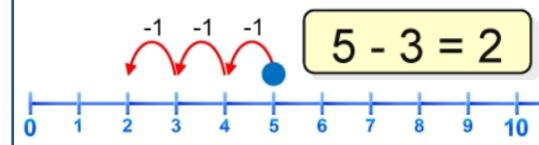
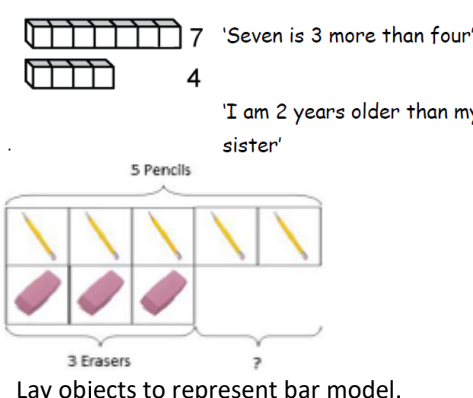
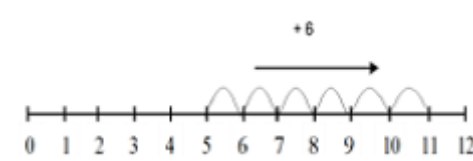
Y3

ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers.	Model using Dienes or numicon  Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and pv counters. 	Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line 	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ Start by partitioning the numbers before formal column to show the exchange. $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$

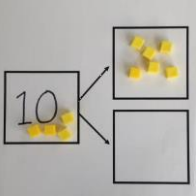
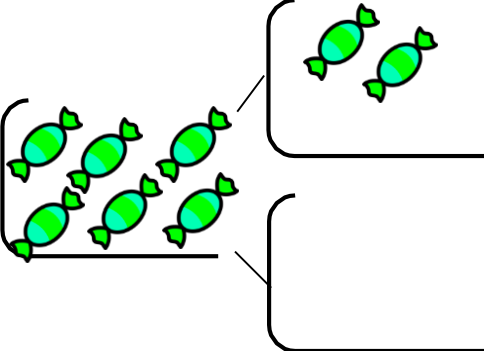
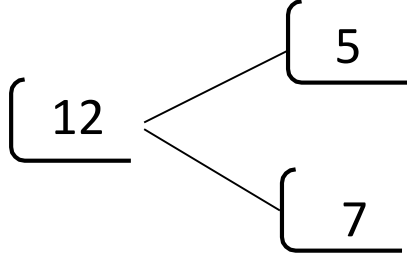
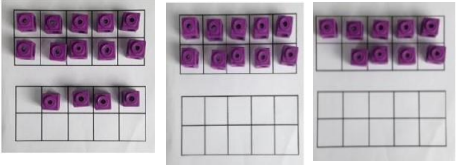
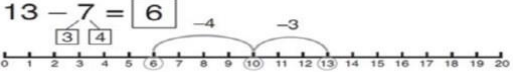
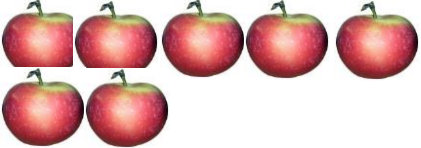
Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using pv grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	  Insert zeros for place holders.

Y4-6 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away.  $6 - 4 = 2$ $4 - 2 = 2$	 $15 - 3 = 12$ Cross out drawn objects to show what has been taken away.	$7 - 4 = 3$ $16 - 9 = 7$
Counting back	 Move objects away from the group, counting backwards. Move the beads along the bead string as you count backwards.	 $5 - 3 = 2$ Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  7 'Seven is 3 more than four' 4 'I am 2 years older than my sister' 5 Pencils 3 Erasers ? Lay objects to represent bar model.	Count on using a number line to find the difference.  +6	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?

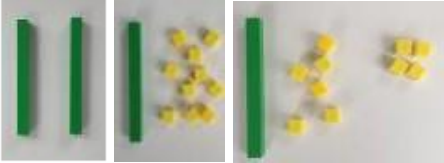
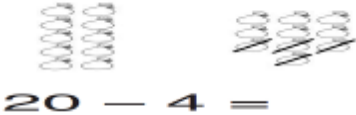
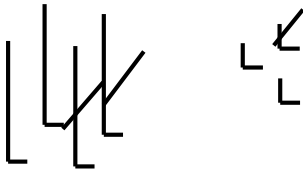
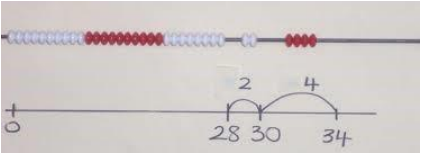
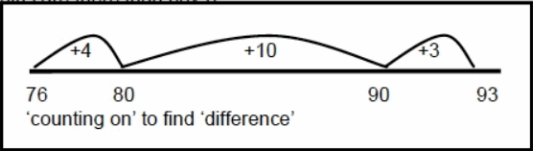
Y1

SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the parts, what's the other part? $10 - 6 = 4$	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	$14 - 9$  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	$13 - 7$  Jump back 3 first, then another 4. Use ten as the stopping point.	$16 - 8$ How many do we take off first to get to 10? How many left to take off?
Bar model	 $5 - 2 = 3$		 $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$

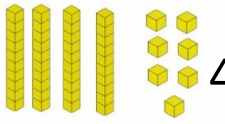
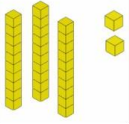
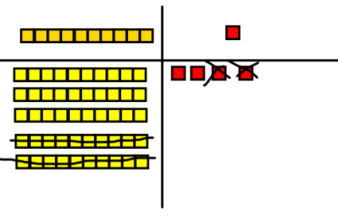
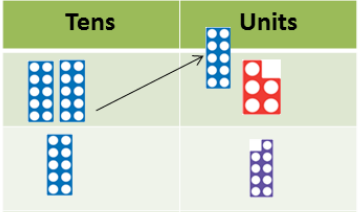
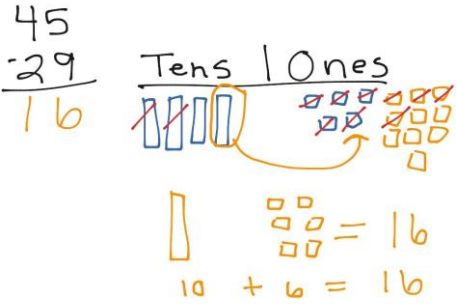
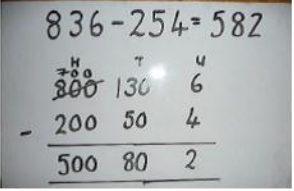
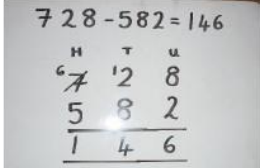
Y1

SUBTRACTION -

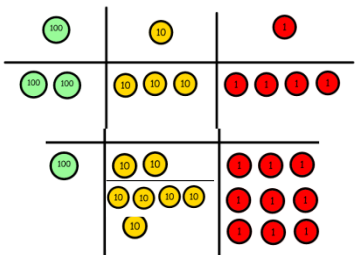
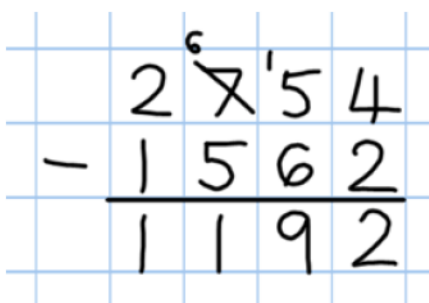
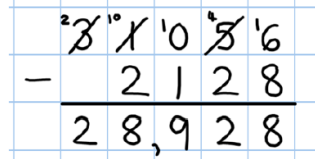
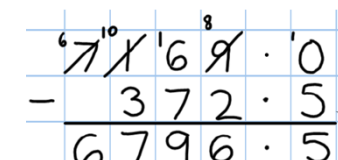
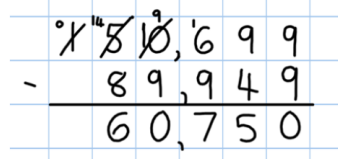
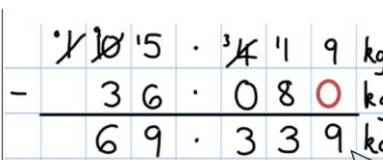
Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	 $20 - 4 = 16$	$20 - 4 = 16$
Partitioning to subtract without regrouping. <i>'Friendly numbers'</i>	$34 - 13 = 21$  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  $43 - 21 = 22$	$43 - 21 = 22$
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 $34 - 28$ Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	$93 - 76 = 17$

Y2

SUBTRACTION -

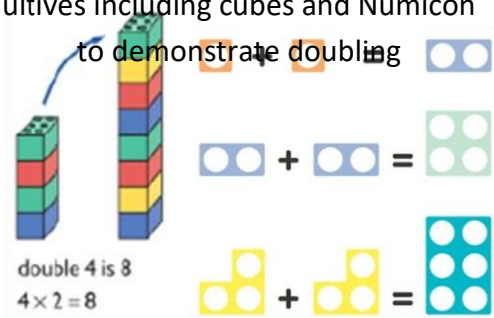
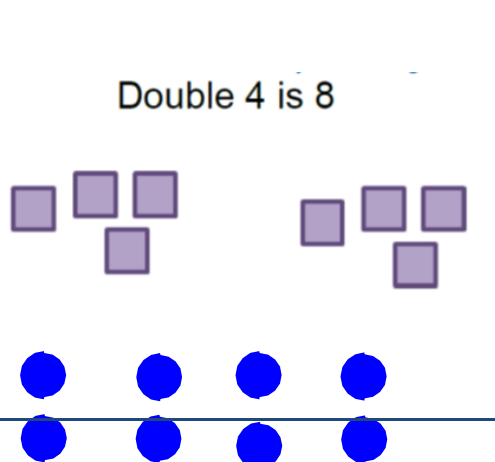
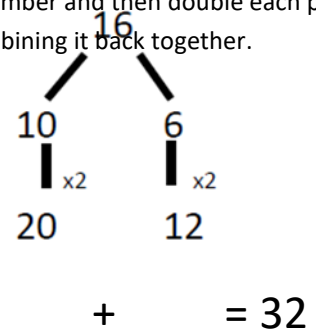
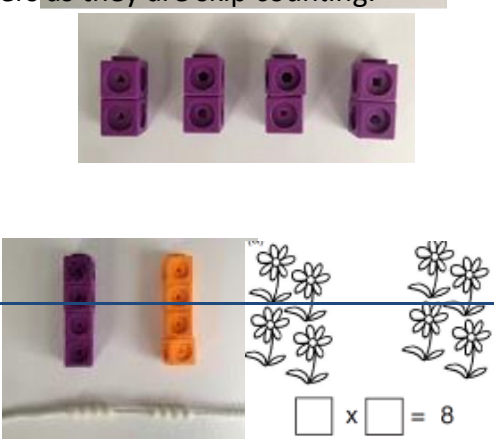
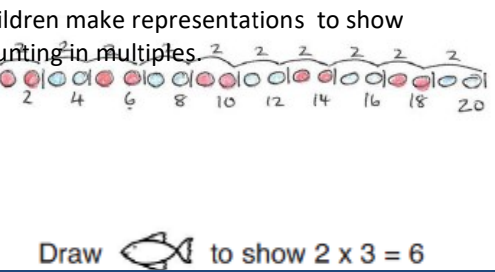
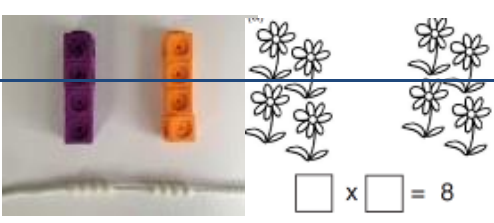
Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 $47 - 32$  Use base 10 or Numicon to model	 Draw representations to support understanding	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

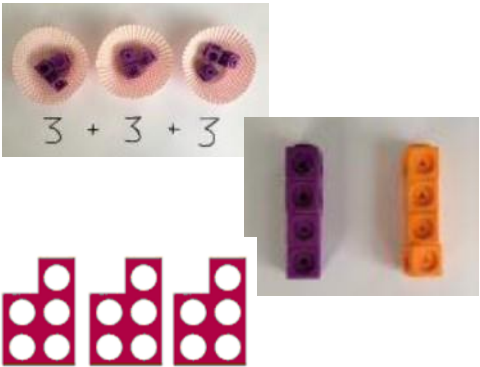
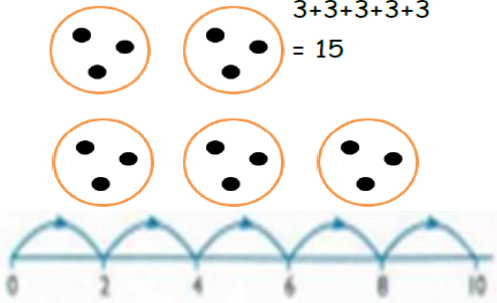
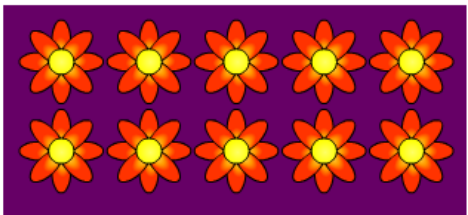
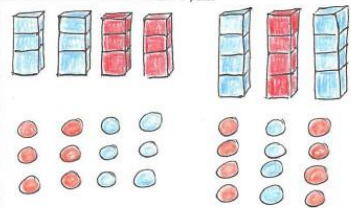
Y3 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. <i>Introduce decimal subtraction through context of money</i>	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. <i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i>	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for place-holders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

Y4-6

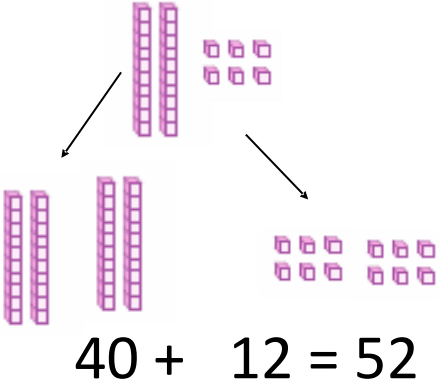
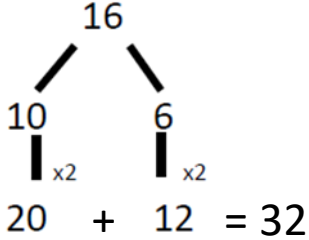
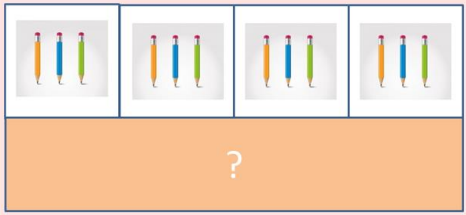
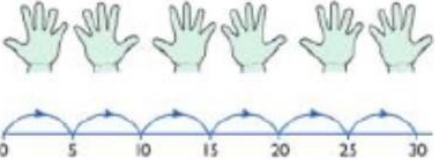
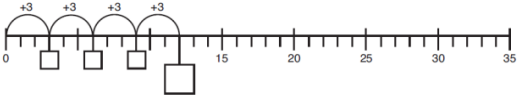
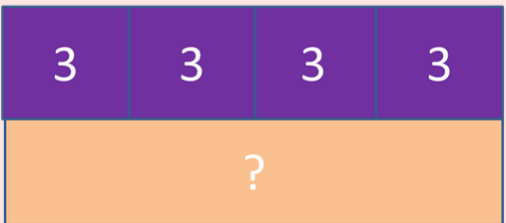
SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.  double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each part before recombining it back together.  $20 + 12 = 32$
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting. 	Children make representations to show counting in multiples.  Draw  to show $2 \times 3 = 6$	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	 $\square \times \square = 8$ Use manipulatives to create equal groups.	Draw and make representations	$2 \times 4 = 8$

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	$3 \times 2 = 6$ $2 \times 5 = 10$

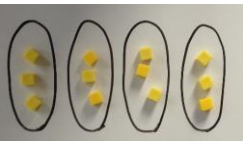
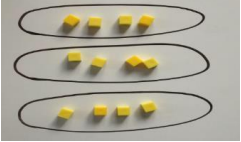
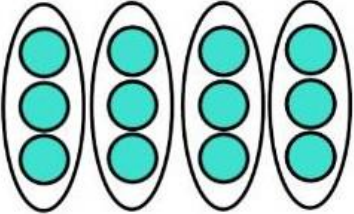
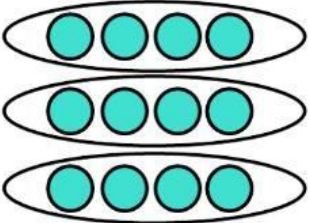
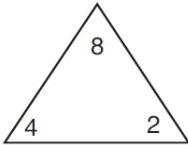
Y2

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  $40 + 12 = 52$	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  $16 \begin{matrix} \swarrow & \searrow \\ 10 & 6 \end{matrix} \begin{matrix} \downarrow \times 2 & \downarrow \times 2 \\ 20 & 12 \end{matrix} \quad 20 + 12 = 32$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 = \square$

Y2

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the Inverse <i>This should be taught alongside division, so pupils learn how they work alongside each other.</i>		 □ × □ = □ □ × □ = □ □ ÷ □ = □ □ ÷ □ = □	$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.

Y3

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method. x 10 3 4 </		

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 100 20 6 Calculations 4 x 126 Fill each row with 126 Add up each column, starting with the ones making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 24 x 3 = 72 20 4 3 00 00 00 60 0000 0000 0000 12 60 72	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. x 30 5 7 210 35 210 + 35 = 245
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642 Hundreds Tens Ones It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	x 300 20 7 4 1200 80 28 The grid method may be used to show how this relates to a formal written method. 59 59 59 59 59 59 59 59 8 x 59 = 8 x 60 - 8 8 x 6 = 48 8 x 60 = 480 480 - 8 = 472 Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	327 x 4 28 80 1200 1308 This may lead to a compact method. 3 2 7 x 4 1 3 0 8 1 2

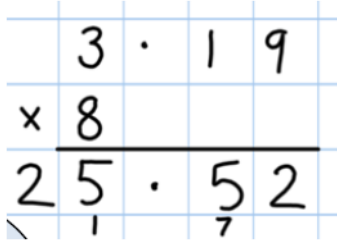
Y4

MULTIPLICATION X

Y5-6

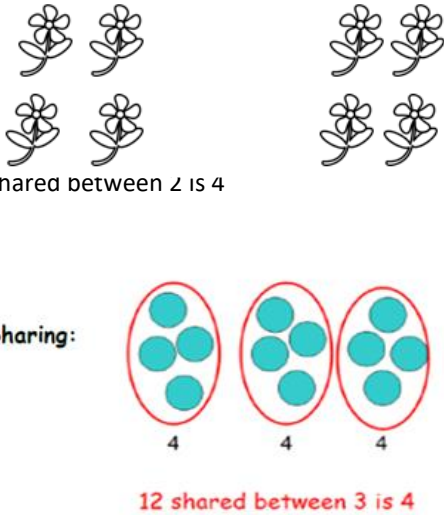
MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract																																																	
Column Multiplication for 3 and 4 digits x 1 digit.	<table><tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table> It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$	Hundreds	Tens	Ones													<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>	x	300	20	7	4	1200	80	28	$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$<table><tr><td></td><td>3</td><td>2</td><td>7</td></tr><tr><td>x</td><td></td><td></td><td>4</td></tr><tr><td></td><td>1</td><td>3</td><td>0</td></tr><tr><td></td><td></td><td>2</td><td>8</td></tr></table> This will lead to a compact method.		3	2	7	x			4		1	3	0			2	8										
Hundreds	Tens	Ones																																																		
x	300	20	7																																																	
4	1200	80	28																																																	
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x			4																																																	
	1	3	0																																																	
		2	8																																																	
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	<table><tr><td></td><td>10</td><td>8</td></tr><tr><td>10</td><td>100</td><td>80</td></tr><tr><td>3</td><td>30</td><td>24</td></tr></table> Continue to use bar modelling to support problem solving		10	8	10	100	80	3	30	24	<table><tr><td></td><td>1</td><td>8</td></tr><tr><td>x</td><td>1</td><td>3</td></tr><tr><td></td><td>5</td><td>4</td></tr><tr><td></td><td>1</td><td>8</td></tr><tr><td></td><td>2</td><td>3</td></tr></table> 18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first <table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>x</td><td></td><td></td><td>1</td><td>6</td></tr><tr><td></td><td>7</td><td>4</td><td>0</td><td>4</td></tr><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td></td><td>1</td><td>9</td><td>7</td><td>4</td></tr></table> (1234 x 6) (1234 x 10)		1	8	x	1	3		5	4		1	8		2	3		1	2	3	4	x			1	6		7	4	0	4		1	2	3	4		1	9	7	4
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	1	9	7	4																																																

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

Y6

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing <i>Use Gordon ITPs for modelling</i>	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 Sharing: 12 shared between 3 is 4	12 shared between 3 is 4

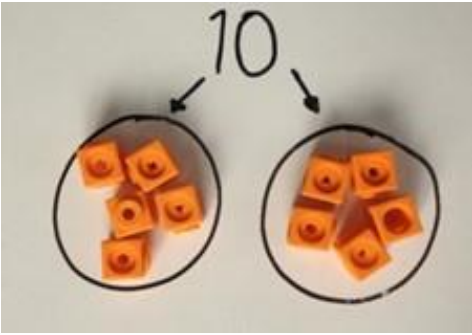
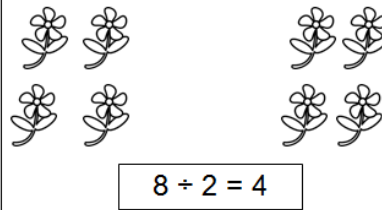
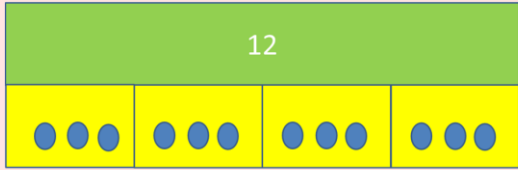
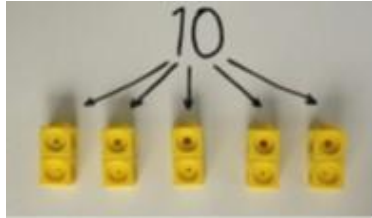
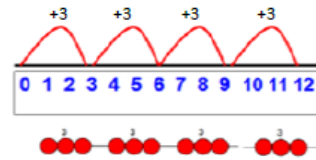
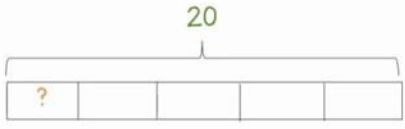
Y1

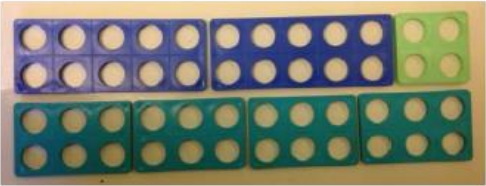
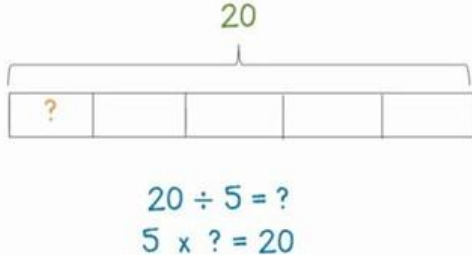
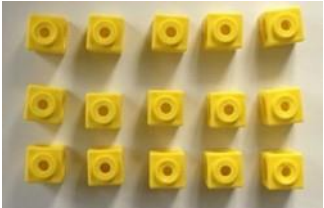
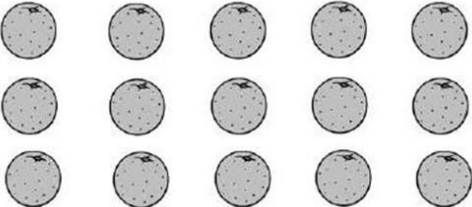
DIVISION ÷

Y2

DIVISION

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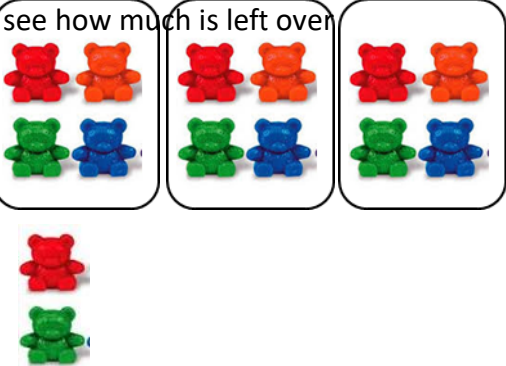
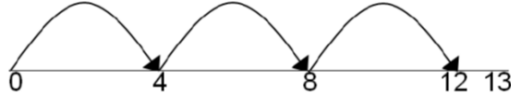
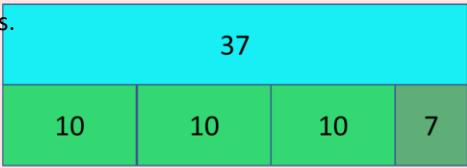
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$ Children use bar modelling to show and support understanding.  $12 \div 4 = 3$	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  $12 \div 3 = 4$ Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 $96 \div 3 = 32$ 	Continue to use bar modelling to aid solving division problems.  $20 \div 5 = ?$ $5 \times ? = 20$	How many groups of 6 in 24? $24 \div 6 = 4$
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Y3

DIVISION

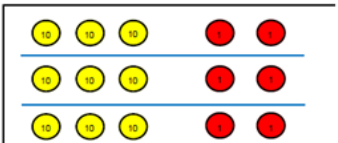
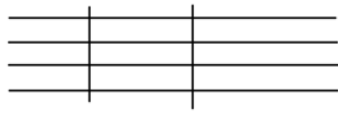
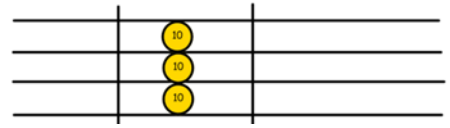
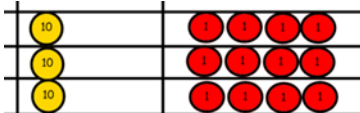
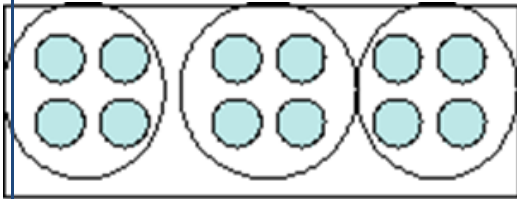
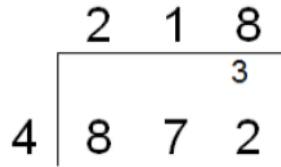
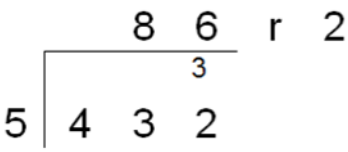
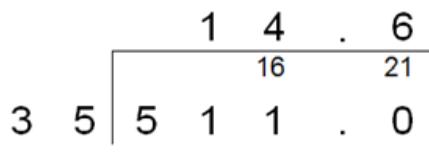
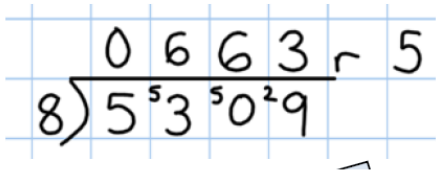
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Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	$14 \div 3 =$ Divide objects between groups and see how much is left over  Example without remainder: $40 \div 5$ Ask "How many 5s in 40?" Example with remainder: $38 \div 6$ For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 8 \text{ fives}$ $0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \quad 30 \quad 35 \quad 40$ $6 + 6 + 6 + 6 + 6 + 6 + 2 = 6 \text{ sixes with a remainder of } 2$ $0 \quad 6 \quad 12 \quad 18 \quad 24 \quad 30 \quad 36 \quad 38$	Complete written divisions and show the remainder using r. $29 \div 8 = 3 \text{ REMAINDER } 5$ $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$ dividend divisor quotient remainder

Y3

Division

÷

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	$96 \div 3$ Tens Units 3 2  Use place value counters to divide using the bus stop method alongside  Calculations $42 \div 3$  $42 \div 3 =$ Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.   We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

Y4-6

Division

Long Division

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Y6

DIVISION



Long Division

Step 1 continued...

$$\begin{array}{r}
 \text{h t o} \\
 061 \\
 4 \overline{) 247} \\
 \underline{-4} \\
 3
 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r}
 \text{th h t o} \\
 0402 \\
 4 \overline{) 1609} \\
 \underline{-8} \\
 1
 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Y6

DIVISION



Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \end{array}$ Two goes into 5 two times, or 5 tens $\div 2 = 2$ whole tens -- but there is a remainder!	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ -4 \\ \hline 1 \end{array}$ To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ -4 \downarrow \\ \hline 18 \end{array}$ Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ -4 \\ \hline 18 \end{array}$ Divide 2 into 18. Place 9 into the quotient.	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ The division is over since there are no more digits in the dividend. The quotient is 29.

Y6

DIVISION



Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \end{array}$ Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ -2 \\ \hline 0 \end{array}$ Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ -2 \downarrow \\ \hline 07 \end{array}$ Next, drop down the 7 of the tens next to the zero.
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ Divide 2 into 7. Place 3 into the quotient.	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ Divide 2 into 18. Place 9 into the quotient.	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ There are no more digits to drop down. The quotient is 139.

Y6

DIVISION

